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## Boundary-Layer Predictions for Small Low-Speed Contractions

James H. Bell\* and Rabindra D. Mehta†  
Stanford University, Stanford, California

### Introduction

CONTRACTION sections form an integral part of all wind tunnels, whether designed for basic fluid flow research or model testing. The main effects of a contraction are to reduce both mean and fluctuating velocity variations to a smaller fraction of the average velocity and to increase the flow mean velocity.<sup>1</sup> The most important single parameter in determining these effects is the contraction ratio  $c$ . Contraction ratios of between 6 and 10 are found to be adequate for most small, low-speed wind tunnels—defined here as tunnels with a test section cross-sectional area of less than about 0.5 m<sup>2</sup> and freestream velocities of less than about 40 m/s.

The wall shape design of a contraction of given area ratio and cross section centers on the production of a uniform and steady stream at its outlet. These conditions generally can be met by making the contraction section sufficiently long. On the other hand, another desirable flow quality, namely a minimum boundary-layer thickness (in a laminar state) at the contraction exit, suggests that the contraction length should be minimized. However, the risk of boundary-layer separation near the two ends of the contraction increases as the length is reduced. In general, the boundary layer is less liable to separate at the contraction exit, due to its reduced thickness caused by passage through the strong favorable pressure gradient. Also, the concave curvature at the contraction inlet has a destabilizing effect on the boundary layer, in contrast to the convex curvature near the exit that has a stabilizing effect.<sup>2</sup> In addition to unnecessary thickening of the boundary layer, separation also generally leads to flow unsteadiness, which cannot be easily eliminated from the test section flow. A design satisfying all criteria will be such that separation is just avoided (implying a minimum acceptable length), and the exit

nonuniformity is equal to the maximum tolerable level for a given application (typically less than 1% variation in mean streamwise velocity outside the boundary layers).

Several papers have been published on the design or choice of contraction wall shapes using a variety of analytical and numerical techniques (see Ref. 3 for a review). Most recent studies have involved the calculation of the wall pressure distributions, using some potential flow numerical scheme, and then the application of a boundary-layer separation criterion based on a critical value of the pressure coefficient. The most popular separation criterion used is that due to Stratford<sup>4</sup> for turbulent boundary-layer separation. We propose in this Note that a boundary layer in a small, low-speed contraction is more likely to start in a laminar state and remain so, for the most part, in passage through it. The normally applied Stratford's criterion for turbulent boundary-layer separation therefore may be too liberal for these designs.

### Computational Approach

A three-dimensional potential flow code (VSAERO) was used to compute the velocity distributions along the contraction walls.<sup>3</sup> VSAERO uses a singularity panel method employing sources and doublets to solve the Laplace equation.

It was hypothesized in this study that for small, low-speed wind tunnels the boundary layers enter the contraction in a laminar state. In most small wind tunnels, the flow entering the contraction comes through a honeycomb and a series of screens (usually at least three). The effect of a screen on a turbulent boundary layer is to significantly reduce its thickness and turbulence stress levels and scales, as shown by Mehta.<sup>5</sup> The results from that investigation showed that a turbulent boundary layer at moderate Reynolds numbers ( $Re_\theta \sim 1600$ ) was effectively relaminarized immediately downstream of the screen. Note that the typical  $Re_\theta$  encountered in small, low-speed settling chambers is likely to be lower by at least an order of magnitude. However, "forced" transition may still occur through either the effects of the Taylor-Görtler instabilities in the regions of concave curvature or a separation bubble. In either case, the strong favorable pressure gradient, encountered in contractions with reasonable area ratios ( $c \sim 6-10$ ), would invariably relaminarize the boundary layer

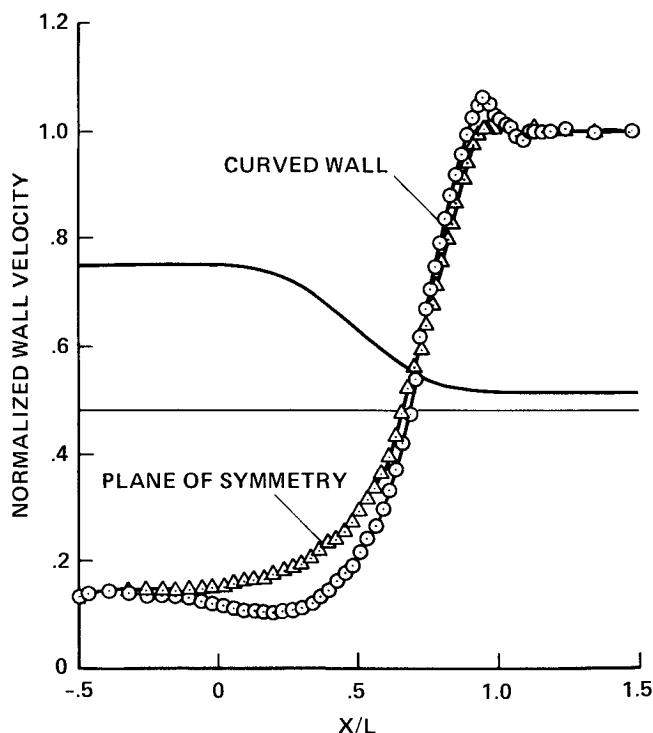


Fig. 1 Typical calculated wall velocities.

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\*Graduate Student, Department of Aeronautics and Astronautics.

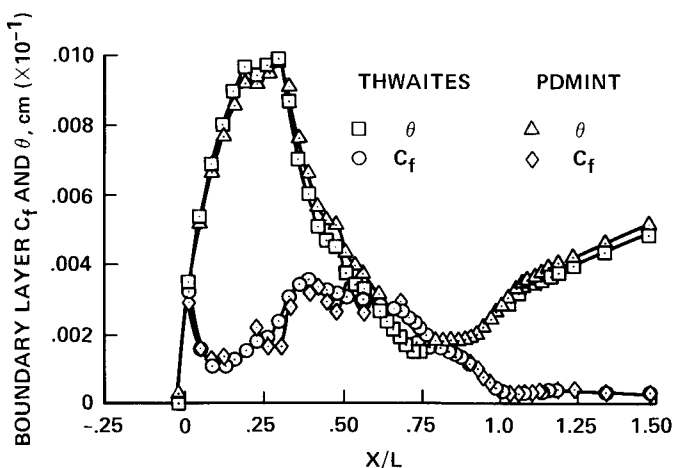
†Research Associate, Department of Aeronautics and Astronautics. Member AIAA.

**Table 1 Details of contractions used for comparisons**

Wind tunnel	Contraction ratio	2-D or 3-D	Inlet height $\times$ width, cm	Exit height $\times$ width, cm	Contraction length, cm	Length-to-height ratio	Exit aspect ratio	Wall contour shape	Splitter plate Y/N
a) Shear layer tunnel (NASA Ames)	10	2-D	38 $\times$ 76	7.6 $\times$ 38	91	1.20	5.0	"Eye" design	Y
b) Mixing layer tunnel (NASA Ames)	7.7	2-D	91 $\times$ 137	91 $\times$ 18	244	0.89	5.14	Fifth-order polynomial	Y
c) Boundary layer tunnel (NASA Ames)	7.5	3-D	120 $\times$ 100	20 $\times$ 80	120	1.0	4.0	Fifth-order polynomial	N
d) Boundary layer tunnel (Imperial College)	9	3-D	114 $\times$ 76	13 $\times$ 76	122	1.07	6.0	"Eye" design	N

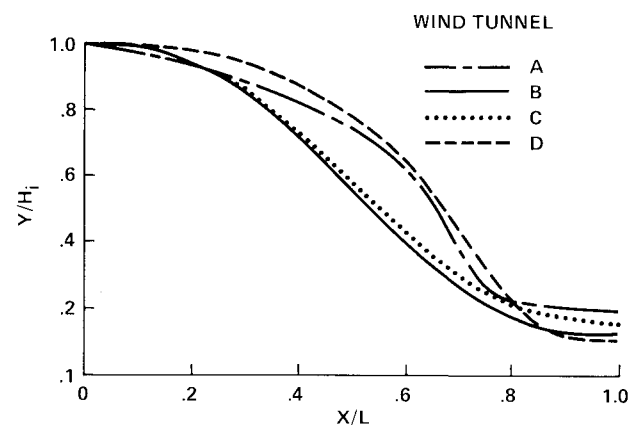
**Table 2 Comparison of momentum thickness results**

Wind tunnel	Test wall	Freestream velocity $U_e$ , m/s	Measured $\theta$ , cm	Predicted $\theta$ , cm	Difference, %
a	Splitter plate low-speed side	10	0.046	0.0425	- 8
	Splitter plate high-speed side	21	0.032	0.0288	- 11
b	Splitter plate low-speed side	9	0.0615	0.0621	1
	Splitter plate high-speed side	15	0.0508	0.0483	- 5
c	Test surface following curved wall	25	0.0325	0.0326	1
d	Test surface following curved wall	16	0.0350	0.0347	- 1

**Fig. 2** Calculations of boundary-layer development through a typical contraction.

soon after. Therefore, the assumption of a laminar boundary layer originating from stagnation conditions at the contraction entrance and remaining laminar throughout was considered to be an adequate approximation for all cases.

The wall velocity data computed by VSAERO were used as boundary conditions for the viscous calculations with stagnation conditions specified at the contraction entry. Two different two-dimensional boundary-layer codes were employed, one a simple integral method solving the momentum integral equation for laminar boundary layers (Thwaites method)<sup>6</sup> and the other a finite-difference technique solving the boundary-layer equations (PDMINT).<sup>7</sup> It is worth noting that neither of these methods accounted for the effects of longitudinal curvature or lateral divergence on boundary-layer structure explicitly. The boundary layers were calculated along the centerline

**Fig. 3** Wall contour shapes of contractions used for verification of the computational scheme ( $L$  is the contraction length and  $H_i$  the inlet height).

of the contraction wall and the plane of symmetry (representing a splitter plate).

### Results and Discussion

A typical example of the wall velocity distributions as computed by VSAERO is given in Fig. 1. Note the presence of regions of adverse pressure gradient on the curved wall near the contraction inlet and outlet indicated by the decrease in wall velocity. The momentum thickness  $\theta$  and skin-friction coefficient  $C_f$  distributions computed using both the boundary-layer programs along the curved wall of the same contraction section are shown in Fig. 2. Initially, the boundary layer grows rapidly in the inlet region where the effects of the first adverse pressure gradient are felt. This is soon overwhelmed by the effects of the strong favorable pressure gradi-

ent, resulting in a rapid reduction in  $\theta$ , before the effects due to the adverse gradient near the outlet take over. The  $C_f$  remains well positive throughout the contraction, thus indicating that separation is not predicted for this particular design. The results from the two techniques are seen to compare favorably, especially for the region near the contraction outlet. Since the program for Thwaites method is relatively easy to obtain and use,<sup>6</sup> all of the boundary-layer data presented below were computed using this method.

In order to validate the proposed computational scheme, the boundary-layer properties in four contractions installed on blower-driven wind tunnels (Table 1) were calculated and compared with actual measurements. The wall shapes for the four contraction sections are plotted in normalized coordinates in Fig. 3. Two of the wall shapes were based on a fifth-order polynomial, while the other two were designed "by eye" following the guidelines given in Ref. 1. Note that for all four contractions investigated, the nondimensional geometric parameters are similar; the contraction ratio is about 8, the length to inlet height ratio is about 1, and the cross-sectional aspect ratio at the exit is about 5.

For all four contraction shapes, the computations predicted an attached boundary layer along the entire length. The measured values of the boundary-layer momentum thickness at the exit of the contractions are compared with those predicted by Thwaites' method in Table 2. The comparisons are made along the wind-tunnel centerline at a short distance (typically less than 15 cm) downstream of the contraction exit. The predicted values of the momentum thickness for all four cases are within about 10% of the measured ones; the typical measurement repeatability was within 1%. The predicted values generally are lower than the measured ones, presumably due to the generation of weak secondary flows along the contraction walls,<sup>8</sup> which tend to thicken the boundary layer along the tunnel centerline. Note that the maximum error between the predictions and experiments occurs for wind tunnel A, which also has the maximum three-dimensionality in the contraction geometry. The agreement between predictions and measurements for all the other boundary-layer properties was comparable and details are given in Ref. 3.

### Conclusions

A scheme is proposed for the prediction of boundary-layer development in small, low-speed contraction sections. The wall pressure distributions, and hence the wall velocity distributions, are first calculated using a three-dimensional potential flow method. Although a panel method was used in this investigation, in principle, any potential flow solver should be acceptable. Once the wall velocities have been obtained, the boundary-layer behavior can be adequately calculated, rather than relying on some separation criterion. For the family of contractions discussed in this Note, the assumption of a laminar boundary layer originating at the contraction entrance and remaining laminar in passage through it seems justified. The measured boundary-layer momentum thicknesses at the exit of four existing contractions, two of which were three-dimensional, were found to lie within 10% of the predicted values, with the predicted values generally lower. The present results indicate that the relatively simple Thwaites method is probably adequate for most purposes. If the prediction accuracy of within 10% on  $\theta$  is acceptable, then the present results also suggest that an iterative process, accounting for the boundary-layer displacement thickness, is not necessary.

### Acknowledgments

This work was supported by the Fluid Dynamics Research Branch, NASA Ames Research Center under Grant NCC-2-294. We would like to thank our colleagues at NASA Ames Research Center for many helpful comments.

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## Motion and Deformation of Very Large Space Structures

Ramesh B. Malla\*

University of Connecticut, Storrs, Connecticut  
and

William A. Nash† and Thomas J. Lardner†

University of Massachusetts, Amherst, Massachusetts

### Introduction

THE effects of various space environmental factors (such as gravitational forces, radiation heating, radiation pressure, space drag, and particle impact) on space structures have long been realized and studied by numerous investigators in past for small-sized space structures (for example, see reference list in Ref. 1). The deployment of very large flexible lightweight structures in space, which have characteristic dimensions of several kilometers operating at altitudes of 200-35,900 km above the Earth's surface, gives rise to a number of new and significant problem areas that may or may not have been associated with smaller systems.

This Note presents a theoretical development of the equations of motion of a very large axially flexible structure orbiting the Earth with planar motion in a general noncircular orbit. With the help of these equations of motion, it is anticipated that effects of many disturbances can be investigated on large space structures' orbital motion, attitude motion, and axial deformation (length). As an application of the equations of motion, the effects of the orbit eccentricity are studied on the coupled orbital, attitude, and axial motion of a large space structure under the influence of the Earth's gravitational forces.

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\*Visiting Assistant Professor, Department of Civil Engineering.

†Professor, Department of Civil Engineering.